

Nonzero U_{e3} , CP violation and leptogenesis in a see-saw type softly broken A_4 symmetric model

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Abstract

We have shown that non-zero U_{e3} is generated in a see-saw type softly broken A_4 symmetric model through a single parameter perturbation in m_D in a single element. We have explored all possible 9 cases to study the neutrino mixing angles considering the best fitted values of Δm_{\odot}^2 and Δm_{atm}^2 with all parameters real. We have extended our analysis for the complex case and demonstrated large low energy CP violation ($J_{CP} \simeq 10^{-2}$) and m_{ee} in addition to mixing and mass pattern. We have also investigated leptogenesis and for a reasonable choice of model parameters compatible with low energy data, WMAP value of baryon asymmetry 6×10^{-10} is obtained for right handed neutrino mass scale $M_0 \simeq 10^{13}$ GeV. We have obtained a relation among the phases responsible for leptogenesis and have shown that those phases also have correlations with low energy CP violating phases.

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1 Introduction

In recent time people are too much interested to find some flavor symmetry in order to generate mass and mixing pattern of fermions. Continuous symmetry like $L_e - L_\mu - L_\tau$ [1], $L_\mu - L_\tau$ [2] symmetry and most popular discrete symmetry, $\mu - \tau$ exchange symmetry ($S_2^{\mu\tau}$) [3] have got some success to describe mass and mixing pattern in leptonic sector. To avoid mass degeneracy of μ and τ under $S_2^{\mu\tau}$ symmetry, E. Ma and G. Rajasekaran in [5] have introduced first time the A_4 symmetry.

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After this paper, a lot of work have done with this symmetry [5]-[44]. After introduction of tri-bi maximal mixing pattern ($\sin \theta_{12} = 1/\sqrt{3}$, $\sin \theta_{23} = -1/\sqrt{2}$, $\sin \theta_{13} = 0$) [4], people have tried to fit this mixing pattern through the A_4 symmetry. In an well motivated extension of the Standard model through the inclusion of A_4 discrete symmetry tri-bi maximal mixing pattern comes out in a natural way in the work of Altarelli and Feruglio [6]. More precisely, the leptonic mixing arises solely from the neutrino sector since the charged lepton mass matrix is diagonal. The model [6] also admits hierarchical masses of the three charged leptons whereas the neutrino masses are quasi-degenerate or hierarchical. Although the model gives rise to $\theta_{13} = 0$ ($U_{e3} = 0$) which is consistent with the CHOOZ-Palo Verde experimental upper bound ($\theta_{13} < 12^\circ$ at 3σ), however, the non-zero and complex value of U_{e3} leads to the possibility to explore CP violation in the leptonic sector which is the main goal of many future short and long baseline experiments. Within the framework of $SU(2)_L \times U(1)_Y \times A_4$ model, non-zero U_{e3} is generated either through the radiative correction [21] or due to the introduction of higher dimensional mass terms [6]. Generation of non zero complex U_{e3} and possibility of non-zero CP violation has been extensively studied in [17] for the proposed model of Altarelli-Feruglio [6] with explicit soft breaking of A_4 symmetry [21].

In the model [6] the authors showed that the tri-bi maximal mixing pattern is also generated naturally in the framework of see-saw mechanism with $SU(2)_L \times U(1)_Y \times A_4$ symmetry. Exact tri-bi maximal pattern forbids at low energy CP violation in leptonic sector. The textures of mass matrices in [6] could not generate lepton asymmetry also. In the present work, we investigate the generation of non-zero U_{e3} through see saw mechanism by considering a small perturbation in m_D , the Dirac neutrino mass matrix, keeping the same texture of the right-handed Majorana neutrino mass matrix as proposed in Ref.[6]. At first, we have studied in detail perturbation of m_D by adding a small parameter at different entries of m_D and see the variations of three mixing angles in terms of other model parameters considering all of them real. We extend our analysis to the complex case for a suitable texture. We study detailed phenomenology of neutrino mass and mixing including CP violation at low energy, neutrinoless double beta decay and leptogenesis. Our approach to get nonzero U_{e3} is minimal as we break A_4 symmetry explicitly by single parameter in single element of m_D . Generation of CP violation at low energy as well as high energy is also minimal as we consider only one parameter complex.

2 The Model of Altarelli Feruglio with See-Saw: light neutrino phenomenology and leptogenesis

We consider the model proposed in [6], which gives rise to diagonal m_D and M_l (the charged lepton mass matrix) along with a competent texture of M_R and after see-saw mechanism and diagonalisation gives rise to tri-bimaximal mixing pattern. The model consists of several scalar

Lepton	$SU(2)_L$	A_4	
$\psi^l(\nu_l, l)$	2	3	
e_R	1	1	
μ_R	1	1''	
τ_R	1	1'	
N_{lR}	1	3	
Scalar			VEV
h_u	2	1	$\langle h_u^0 \rangle = v_u/\sqrt{2}$
h_d	2	1	$\langle h_d^0 \rangle = v_d/\sqrt{2}$
ξ	1	1	$\langle \xi^0 \rangle = u$
ϕ_S	1	3	$\langle \phi_S^0 \rangle = (v_S, v_S, v_S)$
ϕ_T	1	3	$\langle \phi_T \rangle = (v_T, 0, 0)$

Table 1: List of fermion and scalar fields used in this model, $l = e, \mu, \tau$.

fields to generate required vacuum alignment to obtain tri-bimaximal mixing. In Table I., we have listed the scalar fields and their VEV's and representation content under all those symmetries. The model is fabricated in such a way that after spontaneous breaking of A_4 symmetry, the $S_2^{\mu\tau}$ symmetry remains on the neutrino sector and the charged lepton sector is invariant under Z_3 symmetry. Consider the Lagrangian of the model [6],

$$\begin{aligned} \mathcal{L} = & \frac{y_e}{\Lambda}(\phi_T \bar{\psi}_L) e_R h_d + \frac{y_\mu}{\Lambda}(\phi_T \bar{\psi}_L)' \mu_R h_d + \frac{y_\tau}{\Lambda}(\phi_T \bar{\psi}_L)'' \tau_R h_d + f \bar{\psi}_L N_R h_u \\ & + x_A \xi \bar{N}_L^c N_R + x_B \phi_S \bar{N}_L^c N_R + h.c. \end{aligned} \quad (2.1)$$

After spontaneous symmetry breaking, the charged lepton mass matrix comes out diagonal with $m_e = \frac{y_e v_T v_d}{\sqrt{2}\Lambda}$, $m_\mu = \frac{y_\mu v_T v_d}{\sqrt{2}\Lambda}$, and $m_\tau = \frac{y_\tau v_T v_d}{\sqrt{2}\Lambda}$. The neutrino sector gives rise to the following Dirac and Majorana matrices

$$m_D = f \frac{v_u}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad M_R = \begin{pmatrix} A + 2D/3 & -D/3 & -D/3 \\ -D/3 & 2D/3 & A - D/3 \\ -D/3 & A - D/3 & 2D/3 \end{pmatrix} \quad (2.2)$$

where $A = 2x_A u$, $D = 2x_B v_S$. The structure of light neutrino mass matrix can be obtained from see-saw formula:

$$M_\nu = -m_D M_R^{-1} m_D^T = U_{TB} \begin{pmatrix} \frac{-f^2 v_u^2}{2(D+A)} & 0 & 0 \\ 0 & \frac{-f^2 v_u^2}{2A} & 0 \\ 0 & 0 & \frac{-f^2 v_u^2}{2(D-A)} \end{pmatrix} U_{TB}^T \quad (2.3)$$

where,

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}. \quad (2.4)$$

This is clear from Eq.2.3 that U_{TB} is the diagonalising matrix for light neutrino mass matrix M_ν . The form of U_{TB} is in Eq.2.4 which is nothing but the so called tribimaximal mixing matrix. From Eq.2.3 we have the eigenvalues of M_ν :

$$m_1 = -\frac{f^2 v_u^2}{2(D+A)} \quad m_2 = -\frac{f^2 v_u^2}{2A} \quad m_3 = -\frac{f^2 v_u^2}{2(D-A)} \quad (2.5)$$

From Eq.2.4 we have the mixing angles $\sin \theta_{12} = 1/\sqrt{3}$, $\sin \theta_{23} = -1/\sqrt{2}$ and $\sin \theta_{13} = 0$ and from Eq.2.5 we get the solar and atmospheric mass squared differences as

$$\begin{aligned} \Delta m_{\odot}^2 &= m_2^2 - m_1^2 = \frac{m_0^2 k(k+2)}{(1+k)^2} \\ \Delta m_{atm}^2 &= m_3^2 - m_2^2 = \frac{m_0^2 k(2-k)}{(1-k)^2} \end{aligned} \quad (2.6)$$

where $D = kA$, $m_0 = \frac{f^2 v_u^2}{2A}$ and all parameters are real. From the experiments we know Δm_{\odot}^2 is positive and dictates either $k > 0$ or $k < -2$. If $k > 0$, then it has to be small in order to generate small value of Δm_{\odot}^2 provided m_0^2 is not too small as Δm_{\odot}^2 . But small positive k corresponds to same order of magnitude of Δm_{\odot}^2 and Δm_{atm}^2 which is not acceptable according to the experimental results. Now $k > 0$ only acceptable for $m_0^2 \simeq \Delta m_{\odot}^2$ and hierarchy of Δm_{\odot}^2 and Δm_{atm}^2 obtained with the singular nature of Δm_{atm}^2 as in Eq.2.6 near $k \simeq 1$. This corresponds to normal hierarchical mass spectrum. Again for $m_0^2 \gg \Delta m_{\odot}^2$, $k < -2$ is the physical region. This region of k makes $\Delta m_{atm}^2 < 0$ which is so called inverted ordering of neutrino mass pattern. Again $k+2$ should take small value in order to generate small value of Δm_{\odot}^2 . For one complex parameter $D \equiv D e^{i\phi}$, we can write the mass differences in the following form

$$\begin{aligned} \Delta m_{\odot}^2 &= \frac{m_0^2 k(k + 2 \cos \phi)}{1 + k^2 + 2k \cos \phi} \\ \Delta m_{atm}^2 &= \frac{m_0^2 k(2 \cos \phi - k)}{1 + k^2 - 2k \cos \phi}. \end{aligned} \quad (2.7)$$

In the complex case, positivity of Δm_{\odot}^2 can be obtained either with $k > 0$ and $\cos \phi > -k/2$ or with $k < 0$ and $\cos \phi < -k/2$. For the first case with $m_0^2 \simeq \Delta m_{\odot}^2$ and with $\cos \phi \simeq (1+k^2)/2k$ one can have normal hierarchical mass spectrum. For the second case hierarchy will be inverted and $k + 2 \cos \phi$ have to be small. In both case k should take the value such that the $1 \geq \cos \phi \geq -1$ range also satisfy. The mixing pattern is tri-bi maximal Eq.2.4 and it is independent to the fact

whether the parameters are real or complex. In this mixing pattern $U_{e3} = 0$ and non-zero complex U_{e3} is a basic requirement to see the non-zero Dirac CP violation.

Now we concentrate on the issue of leptogenesis in this model. The decay of right handed heavy Majorana neutrinos to lepton(charged or neutral) and scalar(charged or neutral) generate non-zero lepton asymmetry if i) C and CP are violated, ii) lepton number is violated and iii) decay of right handed neutrinos are out of equilibrium. We are in the energy scale where A_4 symmetry is broken but the SM gauge group remains unbroken. So, the higgs scalars both charged and neutral are physical. The CP asymmetry of decay is characterized by a parameter ε_i which is defined as

$$\varepsilon_i = \frac{\Gamma_{N_i \rightarrow l^- \phi^+, \nu_l \phi^0} - \Gamma_{N_i \rightarrow l^+ \phi^-, \nu_l^c \phi^{0*}}}{\Gamma_{N_i \rightarrow l^- \phi^+, \nu_l \phi^0} + \Gamma_{N_i \rightarrow l^+ \phi^-, \nu_l^c \phi^{0*}}}. \quad (2.8)$$

Spontaneous A_4 symmetry breaking generates right handed neutrino mass and the mass matrix M_R obtained is shown in Eq. 2.2. We need to diagonalize M_R in order to go into the physical basis (mass basis) of right handed neutrino. This form of M_R gives the diagonalising matrix in the tri-bi maximal form U_{TB} in Eq.2.4:

$$U_{TB}^\dagger M_R U_{TB}^* = \text{diag}(M_1, M_2, M_3) = \text{diag}(A + D, A, D - A), \quad (2.9)$$

however, the eigenvalues are not real. We need to multiply one diagonal phase matrix U_P with U_{TB} . Hence, diagonalising matrix $V = U_{TB} U_P$ relates the flavor basis to eigen basis of right handed neutrino:

$$N_{lR} = \sum_{i=1}^3 V_{li}^* N_{iR}. \quad (2.10)$$

In this basis the couplings of N_R with leptons and scalars are modified and it will be:

$$m'_d = m_d V^*. \quad (2.11)$$

At the tree level there are no asymmetry in the decay of right handed neutrinos. Due to the interference between tree level and one loop level diagrams, the asymmetry is generated. There are vertex diagram and self energy diagram to contribute to the asymmetry [47, 48]. The vertex contribution is :

$$\varepsilon_i^V = \frac{1}{4\pi v_u^2 h_{ii}} \sum_{j \neq i} \text{Im}(h_{ij}^2) \times \left[\sqrt{x_{ij}} \left\{ 1 - (1 + x_{ij}) \ln\left(1 + \frac{1}{x_{ij}}\right) \right\} \right] \quad (2.12)$$

and the self energy part is :

$$\varepsilon_i^S = \frac{1}{4\pi v_u^2 h_{ii}} \sum_{j \neq i} \text{Im}(h_{ij}^2) \times \frac{\sqrt{x_{ij}}}{1 - x_{ij}}. \quad (2.13)$$

where $x_{ij} = M_j^2/M_i^2$ and

$$h = m_D'^{\dagger} m_D'. \quad (2.14)$$

The key matrix, whose elements are necessary to calculate leptogenesis, is h . In this model m_D is diagonal and proportional to identity. Hence, h matrix is real diagonal and it is also proportional to identity matrix and it is independent of the form of V . The terms for decay asymmetry generated by “i” th generation of right handed neutrino N_{Ri} for both vertex and self energy contributions are proportional to $\text{Im}(h_{ij}^2)$ (where $j \neq i$) as in Eq.2.12 and Eq. 2.13. All off-diagonal elements of h are zero. So, decay of all three generation of right handed Majorana neutrinos could not generate lepton asymmetry. So, in this model of A_4 symmetry tri-bi maximal mixing pattern is not compatible with the low energy Dirac CP violation as well as high energy CP violation. In order to obtain non-zero θ_{13} , low energy Dirac CP violation and leptogenesis we need to break the A_4 symmetry through not only spontaneously but also explicitly introducing some soft A_4 symmetry breaking (soft in the sense that the breaking parameter is small to consider A_4 as an approximate symmetry) terms in the Lagrangian.

3 Explicit A_4 symmetry breaking and real parameter analysis

We consider minimal breaking of A_4 symmetry through a single parameter in a single element of m_D keeping M_R unaltered as

$$\mathcal{L}_{A_4\text{breaking}} = f \epsilon \bar{\psi}_{\alpha L}^1 N_{\beta R} h_u. \quad (3.1)$$

We introduce the breaking by small dimensionless parameter ϵ to the (α, β) element of Dirac type Yukawa term for neutrino. After spontaneous $SU(2)_L \times U(1)_Y$ symmetry breaking it modifies only one element (α, β) of m_D of neutrino. There are nine possibilities to incorporate the breaking parameter ϵ in m_D . We know that after spontaneous A_4 symmetry breaking, a residual $S_2^{\mu\tau}$ symmetry appears in neutrino sector. There is a special feature of $S_2^{\mu\tau}$ symmetry which ensures one 0 and one maximal $\pi/4$ mixing angles. There is one task to check whether our newly introduced explicit breaking term can break $S_2^{\mu\tau}$ symmetry or not. This is important because we need non-zero θ_{13} . We have seen that in one case out of the nine possibilities, residual $S_2^{\mu\tau}$ symmetry remains invariant. This is $\alpha\beta = 11$ case. In other cases $S_2^{\mu\tau}$ symmetry is broken and one expect non-zero θ_{13} from those cases. Primarily, we consider that all parameters are real. We want to study the mixing pattern and want to see its deviation from tri-bi maximal pattern considering experimental value of mass squared differences of neutrinos. We explore all nine cases including $\alpha\beta = 11$ case. Although $\alpha\beta = 11$ case could not generate non-zero θ_{13} , however, we want to see whether this breaking can reduce the tri-bi maximal value of θ_{12} ($\simeq 35.26^\circ$) to its best fit value ($\simeq 34^\circ$) or not along with the

special feature $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. Here, we explicitly demonstrate the procedure for a single case and for the other cases expressions for eigenvalues and mixing angles are given in Appendix.

(i) Breaking at '22' element : In this case, the structure of m_D is given by

$$m_D = \frac{fv_u}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 + \epsilon & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (3.2)$$

and after implementation of see-saw mechanism keeping the same texture of M_R , three light neutrino mass eigenvalues come out as

$$m_1 = -\frac{f^2 v_u^2}{2(D+A)} \left(1 + \frac{\epsilon}{3}\right) \quad m_2 = -\frac{f^2 v_u^2}{2A} \left(1 + \frac{2\epsilon}{3}\right) \quad m_3 = -\frac{f^2 v_u^2}{2(D-A)} (1 + \epsilon) \quad (3.3)$$

and the three mixing angles come out as

$$\begin{aligned} \sin \theta_{12} &= \frac{1}{\sqrt{3}} - \frac{\epsilon(2A+D)}{3\sqrt{3}D} & \sin \theta_{23} &= -\left[\frac{1}{\sqrt{2}} + \frac{\epsilon}{3} \left(\frac{D}{2\sqrt{2}A} + \frac{\sqrt{2}D}{4A-2D} \right) \right] \\ \sin \theta_{13} &= \frac{\epsilon}{3} \left(\frac{D}{\sqrt{2}A} - \frac{\sqrt{2}D}{4A-2D} \right) \end{aligned} \quad (3.4)$$

Assuming a relationship between the parameters D and A as $D = kA$ we rewrite in a convenient way the above three mixing angles as

$$\sin \theta_{12} = \frac{1}{\sqrt{3}} - \frac{\epsilon(2+k)}{3\sqrt{3}k} \quad \sin \theta_{23} = -\left[\frac{1}{\sqrt{2}} + \frac{\epsilon k(4-k)}{6\sqrt{2}(2-k)} \right] \quad \sin \theta_{13} = \frac{\epsilon k(1-k)}{3\sqrt{2}(2-k)} \quad (3.5)$$

and the mass-squared differences are

$$\begin{aligned} \Delta m_{\odot}^2 &= \frac{m_0^2}{3(1+k)^2} [3k(k+2) + 2\epsilon(2k^2 + 4k + 1)] \\ \Delta m_{atm}^2 &= \frac{m_0^2}{3(1-k)^2} [3k(2-k) - 2\epsilon(2k^2 - 4k - 1)] \end{aligned} \quad (3.6)$$

where $m_0 = f^2 v_u^2 / 2A$. Defining the ratio R in terms of mass-squared differences we get

$$R = \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} = \frac{(k-1)^2 [3k(k+2) + 2\epsilon(2k^2 + 4k + 1)]}{(k+1)^2 [3k(2-k) - 2\epsilon(2k^2 - 4k - 1)]} \quad (3.7)$$

which in turn determines the parameter ϵ as

$$\epsilon = \frac{3k}{2} \frac{[R(2-k)(k+1)^2 - (k-1)^2(k+2)]}{[(k-1)^2(2k^2 + 4k + 1) + R(k+1)^2(2k^2 - 4k - 1)]} \quad (3.8)$$

Similarly, we have evaluated all other possible cases which we have listed in the Appendix.

Now the ϵ parameter is determined in terms of R and k and we substitute it to the expressions for mixing angles. Thus it is possible to explore all three mixing angles θ_{12} , θ_{23} and θ_{13} in terms

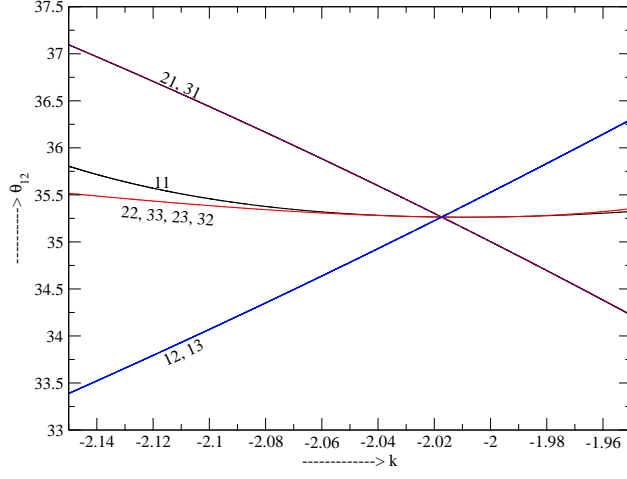


Figure 1: Plot of θ_{12} with respect to k . We keep Δm_{32}^2 and Δm_{21}^2 to their best fit values.

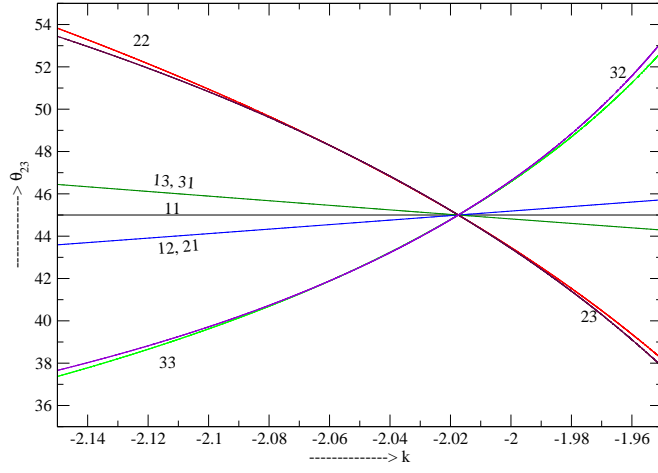


Figure 2: Plot of θ_{23} with respect to k . We keep Δm_{32}^2 and Δm_{21}^2 to their best fit values.

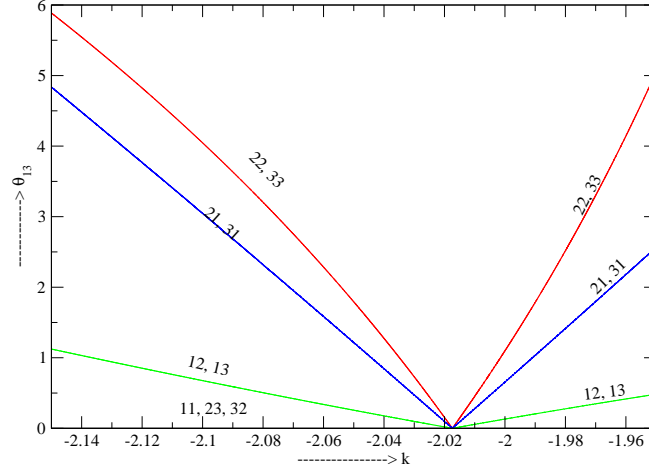


Figure 3: Plot of θ_{13} with respect to k . We keep Δm_{32}^2 and Δm_{21}^2 to their best fit values.

of R and k . Particularly, the deviation from tri-bimaximal mixing depends only on R and k . For the best fit values of the solar and atmospheric mass squared differences ($R \simeq 4 \times 10^{-2}$), we have shown in Fig. 1, Fig. 2 and Fig. 3 the variations of θ_{12} , θ_{23} , θ_{13} versus k , respectively. We have studied all nine possible cases and shown in the plots. First of all, non-zero value of θ_{13} is obtained if we allow A_4 symmetry breaking terms explicitly in any one of the '12, 13, 21, 31, 22, 33' element of the Dirac neutrino mass matrix and those are the cases of our interest. In the present analysis, we have shown that non-zero θ_{13} is generated in a softly broken A_4 symmetric model which leads to deviation from the tri-bimaximal mixing. In A_4 symmetric model θ_{13} is zero because of residual $S_2^{\mu\tau}$ symmetry in neutrino sector after spontaneous breaking of A_4 symmetry. Apart from the explicit breaking of A_4 symmetry at 11 element, $S_2^{\mu\tau}$ is broken for all '12, 13, 21, 31, 22, 33, 23, 32' cases. Furthermore, perturbation around '23', '32' elements also lead to zero value of θ_{13} at the leading order although $S_2^{\mu\tau}$ symmetry is broken, non-zero value is generated if we consider higher order terms of ϵ^2 which are too tiny and hence, discarded from our analysis. We include 11 case for completeness which preserves $S_2^{\mu\tau}$ symmetry and hence generates $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. It only shifts θ_{12} from the tri bimaximal value, but it cannot be able to go towards the best fit value of solar angle, 34° .

If $k = -2$, then we get $\epsilon \propto R$, and thereby, the value of θ_{13} is very small also θ_{12} will hit the exact tri-bimaximal value in some cases. The effect of variation on the mixing angles around $k = -2$ are asymmetric. For some cases (for example 23, 32) θ_{23} changes very fast in the $k > -2$

region. So, we explore the mixing angles with the range $-2.15 \leq k \leq -1.95$. We choose the most feasible cases in which perturbation is applied around '12', '13' elements, because in those cases, variation of k encompasses the best-fit values of θ_{12} and θ_{23} . Although, in the '21', '31' cases, the value of θ_{23} touches the best-fit value $\pi/4$, however, θ_{12} far apart from the best-fit value. In order to achieve large θ_{13} , we have to choose the '21', '31' cases, but we have to allow the variation of θ_{12} around as large as 37.2° . In case of '22', '33', the structure of m_D is still diagonal and also we can get larger θ_{13} (upto 6°) and also θ_{12} is within $1\sigma(36^\circ)$, however, θ_{23} will reach $3\sigma(54^\circ)$ value.

In summary, we have shown that non-zero θ_{13} is generated in a A_4 symmetric model which leads to deviation from the 'tri-bimaximal' mixing through see-saw mechanism due to the incorporation of an explicit A_4 symmetry breaking term in m_D . The breaking is incorporated through a single parameter ϵ and we have investigated the effect of such breaking term in all nine elements of m_D . Some of them generates still zero value of θ_{13} and rest of the others generated non-zero θ_{13} . We expressed all three mixing angles in terms of one model parameter and showed the variation of all three mixing angles with the model parameter k . We find breaking through '12' and '13' elements of m_D are most feasible in view of recent neutrino experimental results.

4 Complex extension: Light neutrino phenomenology and Leptogenesis

In this section, we consider one of the parameter is complex and out of all nine cases as mentioned earlier, we investigate one suitable case arises due to '13' element perturbation. This is one of the suitable positions of breaking justified from real analysis. Again this extension is minimal to generate non-zero CP violation because we consider only one parameter complex. We take D as complex: $D \equiv De^{i\phi}$. Hence, the form of m_D and M_R under explicit A_4 symmetry breaking with complex extension are :

$$m_D = f \frac{v_u}{\sqrt{2}} \begin{pmatrix} 1 & 0 & \epsilon \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad M_R = \begin{pmatrix} A + (2De^{i\phi})/3 & -(De^{i\phi})/3 & -(De^{i\phi})/3 \\ -(De^{i\phi})/3 & (2De^{i\phi})/3 & A - (De^{i\phi})/3 \\ -(De^{i\phi})/3 & A - (De^{i\phi})/3 & (2De^{i\phi})/3 \end{pmatrix}. \quad (4.1)$$

4.1 Light neutrino phenomenology

Using the see-saw mechanism we get the light neutrino mass matrix as

$$M_\nu = -m_D M_R^{-1} m_D^T = \frac{-f^2 v_u^2}{2} \times \quad (4.2)$$

$$U_{TB} \begin{pmatrix} \frac{1}{De^{i\phi}+A} - \frac{2\epsilon}{3(De^{i\phi}+A)} - \frac{2De^{i\phi}(2A+De^{i\phi})\epsilon^2}{9A(A^2-D^2e^{i2\phi})} & \frac{(A+2De^{i\phi})\epsilon}{3\sqrt{2}(De^{i\phi}+A)} - \frac{\sqrt{2}De^{i\phi}(2A+De^{i\phi})\epsilon^2}{9A(A^2-D^2e^{i2\phi})} & \frac{-\epsilon}{\sqrt{3}(A-De^{i\phi})} \\ \frac{(A+2De^{i\phi})\epsilon}{3\sqrt{2}(De^{i\phi}+A)} - \frac{\sqrt{2}De^{i\phi}(2A+De^{i\phi})\epsilon^2}{9A(A^2-D^2e^{i2\phi})} & \frac{1}{A} + \frac{2\epsilon}{3A} - \frac{De^{i\phi}(2A+De^{i\phi})\epsilon^2}{9A(A^2-D^2e^{i2\phi})} & \frac{-\epsilon}{\sqrt{6}(A-De^{i\phi})} \\ \frac{-\epsilon}{\sqrt{3}(A-De^{i\phi})} & \frac{-\epsilon}{\sqrt{6}(A-De^{i\phi})} & \frac{1}{De^{i\phi}-A} \end{pmatrix} U_{TB}^T.$$

We need to diagonalize the M_ν to obtain the masses and mixing angles. The eigenvalues are same as we have in the real case and only difference is that the D is complex now. We explicitly write down the complex phase in the mass matrix. The obtained eigenvalues are :

$$m_1 = -\frac{f^2 v_u^2}{2(De^{i\phi}+A)} \left(1 - \frac{2\epsilon}{3}\right) \quad m_2 = -\frac{f^2 v_u^2}{2A} \left(1 + \frac{2\epsilon}{3}\right) \quad m_3 = -\frac{f^2 v_u^2}{2(De^{i\phi}-A)} \quad (4.3)$$

where we keep terms upto first order in ϵ . Now with $D = kA$, $m_0 = f^2 v_u^2/2A$ and keeping term upto first order in ϵ we get the three neutrino mass squared as

$$|m_1|^2 = \frac{m_0^2(9-12\epsilon)}{9(1+k^2+2k\cos\phi)} \quad |m_2|^2 = \frac{m_0^2(9+12\epsilon)}{9} \quad |m_3|^2 = \frac{m_0^2}{1+k^2-2k\cos\phi}. \quad (4.4)$$

Using those expressions we get the mass squared differences and their ratio which are,

$$\Delta m_{\odot}^2 = |m_2|^2 - |m_1|^2 = \frac{m_0^2\{9k(k+2\cos\phi) + 12\epsilon(2+k^2+2k\cos\phi)\}}{9(1+k^2+2k\cos\phi)} \\ \Delta m_{atm}^2 = |m_3|^2 - |m_2|^2 = \frac{m_0^2\{9k(2\cos\phi-k) - 12\epsilon(1+k^2-2k\cos\phi)\}}{9(1+k^2-2k\cos\phi)} \quad (4.5)$$

and

$$R = \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} = \frac{1+k^2-2k\cos\phi}{1+k^2+2k\cos\phi} \left[\frac{9k(k+2\cos\phi) + 12\epsilon(2+k^2+2k\cos\phi)}{9k(2\cos\phi-k) - 12\epsilon(1+k^2-2k\cos\phi)} \right]. \quad (4.6)$$

The mixing angles are obtained from diagonalisation of M_ν . We solve the equations of the form $M_\nu |m_i\rangle^* = m_i |m_i\rangle$. These $|m_i\rangle$ will give the columns of the diagonalising unitary matrix U . Throughout our calculation we assume that breaking parameter ϵ is small. We have the nonzero U_{13} which is proportional to ϵ . So, the values of U_{12} and U_{23} will give the solar and atmospheric mixing angles, respectively. The expressions for the mixing angles come out as

$$\sin\theta_{12} = |U_{12}| = \frac{1}{\sqrt{3}} + \frac{\epsilon}{3\sqrt{3}} \times \frac{2+4k^2+2k^4+9k\cos\phi+10k^2\cos^2\phi+9k^3\cos\phi}{k(1+k^2+2k\cos\phi)(k+2\cos\phi)} \quad (4.7)$$

$$\sin\theta_{23} = |U_{23}| = \frac{1}{\sqrt{2}} + \frac{\epsilon k}{6\sqrt{2}\cos\phi} \times \frac{k^3+k+2k\cos^2\phi-3k^2\cos\phi-\cos\phi}{k^3+k+4k\cos^2\phi-4k^2\cos\phi-2\cos\phi} \quad (4.8)$$

$$\sin\theta_{13} = |U_{13}| = \frac{\epsilon}{3\sqrt{2}\cos\phi(k^3+k+4k\cos^2\phi-4k^2\cos\phi-2\cos\phi)} \times \\ \left[(k^4+k^2-3k^3\cos\phi-k\cos\phi+6k\cos^3\phi-3\cos^2\phi-k^2\cos^2\phi)^2 \right. \\ \left. + \sin^2\phi(k^3+k+6k\cos^2\phi-5k^2\cos\phi-3\cos\phi)^2 \right]^{1/2}. \quad (4.9)$$

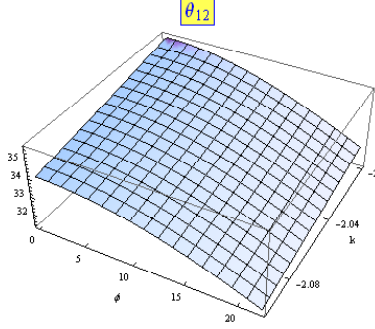


Figure 4: Plot of θ_{12} with respect to k and ϕ for the breaking of A_4 in '13' element of m_D . We keep Δm_{32}^2 and Δm_{21}^2 to their best fit values.

From the expression of mixing angles it is clear that the deviations from tri-bi maximal are first order in ϵ . The independent parameters in this model are A , D , f , ϵ and ϕ . Alternatively the independent parameters are M_0 , m_0 , k , ϵ and ϕ (where $M_0 = A, m_0 = f^2 v_u^2 / 2A$, $k = D/A$). In the above analysis of light neutrino mass and mixing, scale M_0 did not appear explicitly. We have four well measured observable which are Δm_{\odot}^2 , Δm_{atm}^2 , θ_{12} and θ_{23} , and, thus, in principle it is possible to determine four parameters m_0 , k , ϵ and ϕ and we are able to predict the other less known observable such as angle θ_{13} , CP violating parameter J_{CP} etc. It is difficult to get inverse relations of those observable. From the expression of R in Eq. 4.6 we easily obtain the expression for ϵ as

$$\epsilon = \frac{3k [R(2 \cos \phi - k)(1 + k^2 + 2k \cos \phi) - (2 \cos \phi + k)(1 + k^2 - 2k \cos \phi)]}{4 [R(1 + k^2 - 2k \cos \phi)(1 + k^2 + 2k \cos \phi) + (2 + k^2 + 2k \cos \phi)(1 + k^2 - 2k \cos \phi)]}. \quad (4.10)$$

Now using the relation of Δm_{atm}^2 with the parameters Eq. 4.5 we get the expression for m_0^2 :

$$m_0^2 = \frac{9(1 + k^2 - 2k \cos \phi) \Delta m_{atm}^2}{9k(2 \cos \phi - k) - 12\epsilon(1 + k^2 - 2k \cos \phi)}. \quad (4.11)$$

where ϵ is in the form of Eq. 4.10. Thus, ϵ and m_0^2 depend on the parameters k , ϕ and experimentally known R . Extraction of k and ϕ from other two known mixing angles is little bit difficult. Rather

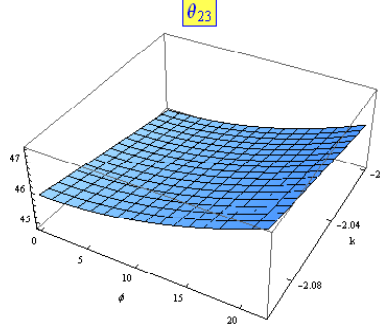


Figure 5: Plot of θ_{23} with respect to k and ϕ for the breaking of A_4 in '13' element of m_D . We keep Δm_{32}^2 and Δm_{21}^2 to their best fit values.

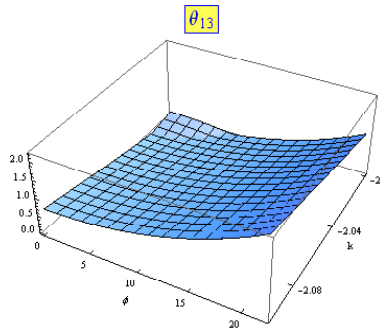


Figure 6: Plot of θ_{13} with respect to k and ϕ for the breaking of A_4 in "13" element of m_D . We keep Δm_{32}^2 and Δm_{21}^2 to their best fit values.

we have plotted θ_{12} and θ_{23} with respect to k and ϕ and obtain the restriction on the parameter space of k and ϕ . From the expression of $\sin \theta_{12}$ in Eq. 4.7 we are seeing that there is a factor $k + 2 \cos \phi$ in the denominator. For $k > -2$ there will be a ϕ for which the quantity $k + 2 \cos \phi$ becomes zero. Hence, we should keep $k < -2$. Again the factor $k + 2 \cos \phi$ should be small to ensure that ϵ is also small. It justifies our whole analysis because we consider first order perturbation as we considered symmetry A_4 remains approximate. We consider the range $-2.1 < k < -2.0$ and $0 < \phi < 22^\circ$. From Fig. 4 we see that θ_{12} changes from the tri-bi maximal value 35.6° to 31° . Near $\phi = 10^\circ$ it crosses the best fit value 34° . We have plotted θ_{23} in Fig. 5. The variation of θ_{23} is from 47° to 44.8° for the same range of k and ϕ . The best fit value 45° is remain within range of variation and it is in the low k low ϕ region. The plot of θ_{13} is in Fig. 6. Value of θ_{13} remains within 2° for the same range of k and ϕ and the model predicts θ_{13} is very small but non-zero. Question may arise whether such small value of θ_{13} can generate observable CP violation or not.

Keeping all those constraints in view next we explore the parameter space of CP violation parameter J_{CP} . The parameter J_{CP} defined as [45]

$$J_{CP} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta_{CP} = \frac{\text{Im}[h'_{12}h'_{23}h'_{31}]}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2} \quad (4.12)$$

where $h' = M_\nu M_\nu^\dagger$, δ_{CP} is Dirac phase. This J_{CP} is associated with CP violation in neutrino oscillation and is directly related to Dirac phase of mixing matrix. The from Eq. (4.3) we can express M_ν matrix in terms of ϵ and m_0 and k and ϕ . With the expressions for ϵ and m_0 we can easily obtain the M_ν and hence $h' = M_\nu M_\nu^\dagger$ completely in terms of k and ϕ . Hence, similar to the mixing angles J_{CP} will be also only function of k and ϕ . We have plotted J_{CP} in Fig. 7 where the values are normalized by a factor 10^{-3} . For the same range of k and ϕ the model predicts J_{CP} up to the order of 10^{-2} which is appreciable to observe through the forthcoming experiments. Inverting the expression of J_{CP} , the phase δ_{CP} is extracted in terms of k and ϕ and it is plotted in Fig. 8. We see that the value of δ_{CP} is large upto 90° and, thereby, compensates small θ_{13} effect in J_{CP} and makes it observable size. One important discussion we have to make about the range of ϕ and k . One can ask why we are keeping ourselves small range of those parameters where larger ϕ can enhance the θ_{13} and J_{CP} . We have studied that the larger value of ϕ and also k in negative become responsible for breaking the analytic bound $\sin \delta_{CP} < 1$. So, we keep ourselves in shrinked parameter space which keep θ_{12} and θ_{23} in exceptionally good values according to the experiment and also can able to generate observable CP violation instead of small θ_{13} . Another thing we want to point out that negative value of ϕ equally acceptable as far as it is small. It could not change mixing angles because their expressions depend on ϕ through $\cos \phi$ and $\sin^2 \phi$. Only J_{CP} and δ_{CP} will change in sign which are unsettled according to the experiments.

At the end we want to check whether the range of k and ϕ can satisfy the double beta decay

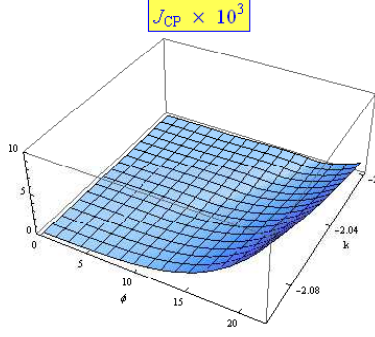


Figure 7: Plot of J_{CP} with respect to k and ϕ for the breaking of A_4 in 13 element of m_D in the unit of 10^{-3} . We keep Δm_{32}^2 and Δm_{21}^2 to their best fit values.

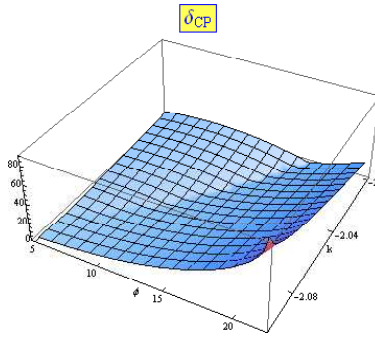


Figure 8: Plot of δ_{CP} with respect to k and ϕ for the breaking of A_4 in “13” element of m_D . We keep Δm_{32}^2 and Δm_{21}^2 to their best fit values.

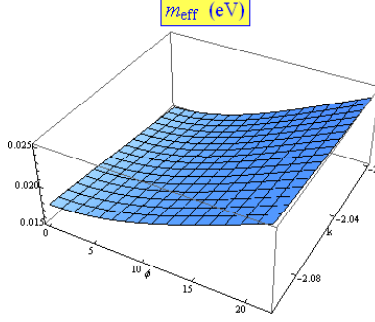


Figure 9: Plot of $m_{\text{eff}} = |(M_\nu)_{ee}|$ with respect to k and ϕ for the breaking of A_4 in “13” element of m_D . We keep Δm_{32}^2 and Δm_{21}^2 to their best fit values.

bound $m_{\text{eff}} = |(M_\nu)_{ee}| \leq 0.89$ eV. In our model expression for this quantity is :

$$m_{\text{eff}} = |(M_\nu)_{ee}| = \frac{m_0}{3} \times \sqrt{\frac{9 + k^2(1 + 2\epsilon)^2 + 6k(1 + 2\epsilon)\cos\phi}{1 + k^2 + 2k\cos\phi}} \quad (4.13)$$

and it will be also only function of k and ϕ . We plot this in Fig. 9 and it remains well below the experimental upper bound.

Again we want to discuss about the mass pattern. Throughout our whole analysis in real as well as complex case we keep Δm_{\odot}^2 and Δm_{atm}^2 to their best fit value and take the negative sign of Δm_{atm}^2 . It corresponds to so called inverted ordering of neutrino mass. It is the feature near $k = -2$. It is necessary to keep $m_0^2 > 0$. Why we so fond of this region of k instead of region $k = 1$ which can give the normal hierarchical mass spectrum. The reason is that the inverted ordering corresponds to the light neutrino mass scale $m_0 \simeq \Delta m_{atm}^2$ where $m_0 \simeq \Delta m_{\odot}^2$ for normally ordered mass spectrum. So, from the point of view of observable CP violation, it is inevitable to choose larger value of m_0 because $J_{CP} \propto m_0^6$. So, inverted hierarchical mass spectrum compatible with the observable CP violation. Now we extend our study of this model to leptogenesis. We want to see whether we can have appreciable leptogenesis compatible with baryon asymmetry for the same parameter space k and ϕ after successful low energy data analysis for the feasible value of scale M_0 .

4.2 Leptogenesis

After successful predictions of low energy neutrino data we want to see whether this model can generate non-zero lepton-asymmetry with proper size and sign to describe baryon asymmetry. We keep the same right handed neutrino mass matrix M_R as before. The change only appear in Dirac type Yukawa coupling and hence in m_D also. The diagonalisation of M_R gives

$$M_R^D = U_{TB}^\dagger M_R U_{TB}^* = \text{diag}(|A + De^{i\phi}|e^{i2\alpha}, |A|e^{i2\lambda}, |De^{i\phi} - A|e^{i2\gamma}) \quad (4.14)$$

and hence the masses of the right handed neutrino are

$$\begin{aligned} M_1 = |A + De^{i\phi}| &= M_0 \sqrt{1 + k^2 + 2k \cos \phi} & M_2 = |A| &= M_0 \\ M_3 &= |De^{i\phi} - A| = M_0 \sqrt{1 + k^2 - 2k \cos \phi} \end{aligned} \quad (4.15)$$

and the phases are

$$\tan 2\alpha = \frac{k \sin \phi}{1 + k \cos \phi} \quad \tan 2\lambda = 0 \quad \tan 2\gamma = \frac{k \sin \phi}{k \cos \phi - 1}. \quad (4.16)$$

The explicit form of diagonalising matrix is

$$V = U_{TB} U_P = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\lambda} & 0 \\ 0 & 0 & e^{i\gamma} \end{pmatrix} \quad (4.17)$$

where the expressions for the phases are given in Eq. (4.16). The m_D matrix is no longer diagonal after explicit breaking of A_4 symmetry. In the mass basis of right handed neutrino the modified Dirac mass term is $m'_D = m_D V^*$. Hence the relevant matrix for describing Leptogenesis is :

$$h = m_D'^\dagger m'_D = \frac{f^2 v_u^2}{2} \begin{pmatrix} 1 - 2\epsilon/3 & \epsilon e^{i(\alpha-\lambda)}/(3\sqrt{2}) & \epsilon e^{i(\alpha-\gamma)}/\sqrt{3} \\ \epsilon e^{-i(\alpha-\lambda)}/(3\sqrt{2}) & 1 + 2\epsilon/3 & \epsilon e^{i(\lambda-\gamma)}/\sqrt{6} \\ \epsilon e^{-i(\alpha-\gamma)}/\sqrt{3} & \epsilon e^{-i(\lambda-\gamma)}/\sqrt{6} & 1 \end{pmatrix}. \quad (4.18)$$

To calculate lepton asymmetry as in Eq. (2.12) and Eq. (2.13) we need to calculate following quantities from matrix h :

$$\begin{aligned} \text{Im}(h_{21}^2) &= -\text{Im}(h_{12}^2) = -\frac{f^4 \epsilon^2 v_u^4 \sin 2(\alpha - \lambda)}{72} = -\frac{f^4 \epsilon^2 v_u^4}{72} \times \frac{k \sin \phi}{\sqrt{1 + k^2 + 2k \cos \phi}} \\ \text{Im}(h_{32}^2) &= -\text{Im}(h_{23}^2) = -\frac{f^4 \epsilon^2 v_u^4 \sin 2(\lambda - \gamma)}{12} = \frac{f^4 \epsilon^2 v_u^4}{24} \times \frac{k \sin \phi}{\sqrt{1 + k^2 - 2k \cos \phi}} \\ \text{Im}(h_{31}^2) &= -\text{Im}(h_{13}^2) = -\frac{f^4 \epsilon^2 v_u^4 \sin 2(\alpha - \gamma)}{12} = \frac{2f^4 \epsilon^2 v_u^4}{12} \times \frac{k \sin \phi}{\sqrt{(1 + k^2)^2 - 4k^2 \cos^2 \phi}}. \end{aligned} \quad (4.19)$$

Calculating $x_{ij} = M_j^2/M_i^2$ from Eq. 4.15, h_{ii} from Eq. (4.18) and taking $\text{Im}(h_{ij}^2)$ from Eq. (4.19) we calculate the self energy part of lepton asymmetry from Eq. (2.13) and vertex part of lepton asymmetry from Eq. (2.12). Adding both we obtain the following decay asymmetry of right handed neutrinos for all three generations

$$\begin{aligned} \varepsilon_1 = & \frac{M_0 m_0 \epsilon^2 k \sin \phi}{72\pi v_u^2 (1 + k^2 + 2k \cos \phi)} \times \left[\frac{1 + 2k^2 + 4k \cos \phi}{k(k + 2 \cos \phi)} - \frac{3(1 + k^2 + 6k \cos \phi)}{k \cos \phi} \right. \\ & \left. - \frac{(2 + k^2 + 2k \cos \phi) \ln(2 + k^2 + 2k \cos \phi)}{1 + k^2 + 2k \cos \phi} + \frac{24(1 + k^2)}{1 + k^2 + 2k \cos \phi} \times \ln \frac{2(1 + k^2)}{1 + k^2 - 2k \cos \phi} \right]. \end{aligned} \quad (4.20)$$

$$\begin{aligned} \varepsilon_2 = & -\frac{M_0 m_0 \epsilon^2 k \sin \phi}{72\pi v_u^2} \times \left[\frac{k^2 + 2k \cos \phi - 1}{k(k + 2 \cos \phi)} + \frac{3(1 - k^2 + 2k \cos \phi)}{k(2 \cos \phi - k)} - (2 + k^2 + 2k \cos \phi) \right. \\ & \left. \times \ln \left\{ \frac{2 + k^2 + 2k \cos \phi}{1 + k^2 + 2k \cos \phi} \right\} - 3(2 + k^2 - 2k \cos \phi) \times \ln \left\{ \frac{2 + k^2 - 2k \cos \phi}{1 + k^2 - 2k \cos \phi} \right\} \right] \end{aligned} \quad (4.21)$$

$$\begin{aligned} \varepsilon_3 = & \frac{M_0 m_0 \epsilon^2 k \sin \phi}{24\pi v_u^2 (1 + k^2 - 2k \cos \phi)} \times \left[\frac{6k \cos \phi - k^2 - 1}{k \cos \phi} + \frac{1 + 2k^2 - 4k \cos \phi}{k(k - 2 \cos \phi)} \right. \\ & \left. - \frac{2 + k^2 - 2k \cos \phi}{1 + k^2 - 2k \cos \phi} \times \ln(2 + k^2 - 2k \cos \phi) - \frac{8(1 + k^2)}{1 + k^2 - 2k \cos \phi} \times \ln \frac{2(1 + k^2)}{1 + k^2 + 2k \cos \phi} \right] \end{aligned} \quad (4.22)$$

CP asymmetry parameters ε_i are related to the leptonic asymmetry parameters through Y_L as [49, 50, 46]

$$Y_L \equiv \frac{n_L - \bar{n}_L}{s} = \sum_i^3 \frac{\varepsilon_i \kappa_i}{g_{*i}} \quad (4.23)$$

where n_L is the lepton number density, \bar{n}_L is the anti-lepton number density, s is the entropy density, κ_i is the dilution factor for the CP asymmetry ε_i and g_{*i} is the effective number of degrees of freedom [51] at temperature $T = M_i$. Value of g_{*i} in the SM with three right handed Majorana neutrinos and one extra Higgs doublet is 116. The baryon asymmetry Y_B produced through the sphaleron transmutation of Y_L , while the quantum number $B - L$ remains conserved, is given by [52]

$$Y_B = \frac{\omega}{\omega - 1} Y_L \quad \text{with} \quad \omega = \frac{8N_F + 4N_H}{22N_F + 13N_H}, \quad (4.24)$$

where N_F is the number of fermion families and N_H is the number of Higgs doublets. The quantity $\omega = 8/23$ in Eq. (4.24) for SM with two Higgs doublet. Now we introduce the relation between Y_B

and η_B , where η_B is the baryon number density over photon number density n_γ . The relation is [53]

$$\eta_B = \left. \frac{s}{n_\gamma} \right|_0 Y_B = 7.0394 Y_B, \quad (4.25)$$

where the zero indicates present time. Now using the relations in Eqs. (4.23, 4.24, 4.25), $\omega = 8/23$ and $g_{*i} = 116$ we have

$$\eta_B = -3.23 \times 10^{-2} \sum_i \varepsilon_i \kappa_i. \quad (4.26)$$

This dilution factor κ_i approximately given by [50, 54, 55]

$$\kappa_i \simeq \frac{0.3}{K_i (\ln K_i)^{3/5}} \quad \text{with} \quad K_i = \frac{\Gamma_i}{H_i}, \quad (4.27)$$

where Γ_i is the decay width of N_i and H_i is Hubble constant at $T = M_i$. Their expressions are

$$\Gamma_i = \frac{h_{ii} M_i}{4\pi v_u^2} \quad \text{and} \quad H_i = 1.66 \sqrt{g_{*i}} \frac{M_i^2}{M_P}, \quad (4.28)$$

where $v_u = v \sin \beta$, $v = 246 \text{ GeV}$ and $M_P = 1.22 \times 10^{19} \text{ GeV}$. Thus we have

$$K_i = \frac{M_P h_{ii}}{1.66 \times 4\pi \sqrt{g_{*i}} v_u^2 M_i}. \quad (4.29)$$

For our model K_1 , K_2 and K_3 are

$$\begin{aligned} K_1 &= \frac{M_P h_{11}}{1.66 \times 4\pi \sqrt{g_{*1}} v_u^2 M_1} = N \times \frac{m_0}{v_u^2} \times \frac{(1 - \frac{2\epsilon}{3})}{(1 + k^2 + 2k \cos \phi)^{1/2}} \\ K_2 &= \frac{M_P h_{22}}{1.66 \times 4\pi \sqrt{g_{*2}} v_u^2 M_2} = N \times \frac{m_0}{v_u^2} \times (1 + \frac{2\epsilon}{3}) \\ K_3 &= \frac{M_P h_{33}}{1.66 \times 4\pi \sqrt{g_{*3}} v_u^2 M_3} = N \times \frac{m_0}{v_u^2} \times \frac{1}{(1 + k^2 - 2k \cos \phi)^{1/2}} \end{aligned} \quad (4.30)$$

where $N = \frac{M_P}{1.66 \times 4\pi \sqrt{g_*}}$, $g_* = g_{*1} \approx g_{*2} \approx g_{*3} \approx 116$. Using Eq. (4.30) into Eq. (4.27) and from the expression of ε_i we can say that apart from logarithmic factor, $\kappa_i \propto v_u^2/m_0$ and $\varepsilon_i \propto m_0/v_u^2$. So, baryon asymmetry will be independent of m_0 and v_u . They only appear through logarithmic factor in κ_i . We consider $\tan \beta \approx 2.5$. Substituting m_0 from Eq. (4.11), ϵ from Eq. (4.10) and considering M_0 some specific value into the expressions of ε_i , κ_i we can have baryon asymmetry as function of k and ϕ only. In Fig. 10, Fig. 11 and Fig. 12 we have plotted η_B as function of k and ϕ in the unit of 10^{-10} for three M_0 values, $M_0 = 2 \times 10^{13} \text{ GeV}$, $M_0 = 5 \times 10^{13} \text{ GeV}$ and $M_0 = 10^{14} \text{ GeV}$ respectively. We have seen that the experimental value of $\eta_B \simeq 6.0 \times 10^{-10}$ is obtainable in our model within the same range of κ and ϕ as in the low energy case. To see more explicitly the baryon asymmetry

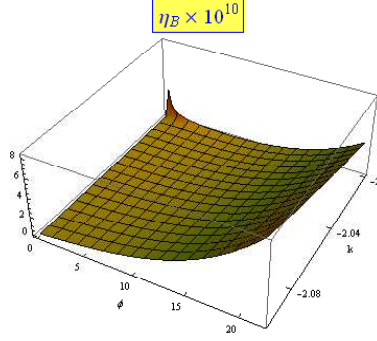


Figure 10: Plot of baryon asymmetry η_B in unit of 10^{-10} with respect to k and ϕ for the breaking of A_4 in “13” element of m_D . We keep Δm_{32}^2 and Δm_{21}^2 to their best fit values and have plotted for mass scale of right handed neutrino $M_0 = 2 \times 10^{13}$ GeV

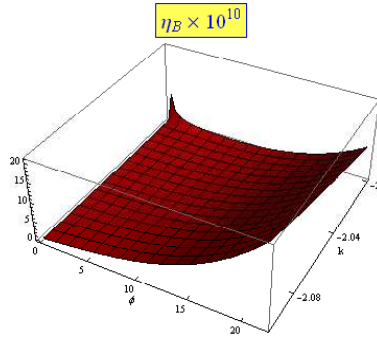


Figure 11: Plot of baryon asymmetry η_B in unit of 10^{-10} with respect to k and ϕ for the breaking of A_4 in “13” element of m_D . We keep Δm_{32}^2 and Δm_{21}^2 to their best fit values and have plotted for mass scale of right handed neutrino $M_0 = 5 \times 10^{13}$ GeV

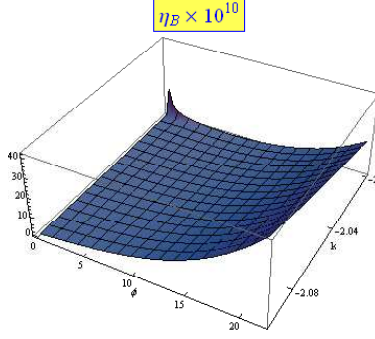


Figure 12: Plot of baryon asymmetry η_B in unit of 10^{-10} with respect to k and ϕ for the breaking of A_4 in “13” element of m_D . We keep Δm_{32}^2 and Δm_{21}^2 to their best fit values and have plotted for mass scale of right handed neutrino $M_0 = 10^{14}$ GeV

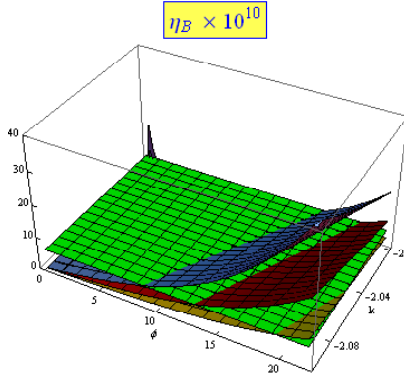


Figure 13: We combine all three plots of baryon asymmetry for three right handed neutrino mass scale along with the WMAP value of baryon asymmetry 6×10^{-10} which is the plane surface.

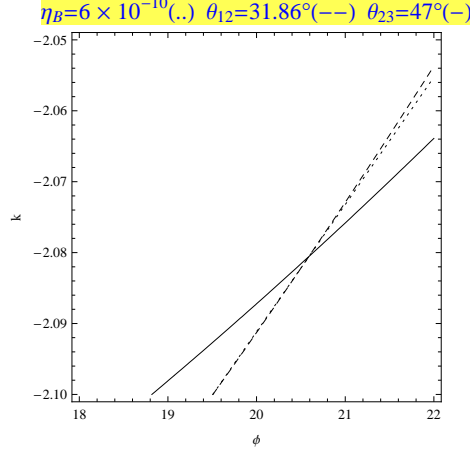


Figure 14: Contour plot of baryon asymmetry η_B , θ_{12} and, θ_{23} in ϕ - k plane for Δm_{32}^2 and Δm_{21}^2 to their best fit values and for mass scale of right handed neutrino $M_0 = 2 \times 10^{13}$ GeV.

plots we combine all three plots along with the observed baryon asymmetry value 6×10^{-10} which corresponds the plane surface in Fig. 13. The observed WMAP value of the baryon asymmetry curve intersect the lower curve (for $M_0 = 2 \times 10^{13}$ GeV) near the boundary of κ and ϕ variation. So, lower value of M_0 could not generate observable baryon asymmetry. In the intersection region $\theta_{12} \simeq 32^\circ$ and $\theta_{23} \simeq 47^\circ$. If we allow that much of variation of θ_{12} and θ_{23} , we can have large low energy CP violation as well as baryon asymmetry with proper size and sign with $M_0 = 2 \times 10^{13}$ GeV which is near the upper bound of right handed neutrino mass scale for generation of lepton as well as baryon asymmetry. If we more relax, we can easily see that the intersection of experimental and theoretical curve for $M_0 = 5 \times 10^{13}$ GeV and $M_0 = 10^{14}$ GeV is in the lower value of k and ϕ where well known neutrino mixing angles are more closer to their best fit value.

Let us give a close look to the plot of η_B in Fig. 10, Fig. 11 and Fig. 12 near very low ϕ and $k \simeq -2$ region. This is the region where η_B become very large. From the expression of ε_1 and ε_2 it is clear that ε_1 and ε_2 both are singular at $k + 2 \cos \phi = 0$. This corresponds to equality of the masses $M_1 = M_2$ or $x_{12} = x_{21} = 1$. This singularity can be avoided considering finite decay width of right handed neutrinos. We can able to maximize ε_1 and ε_2 and hence η_B using resonant condition $M_1 - M_2 \simeq \Gamma_{1,2}/2$. But, as we have already obtained the observed baryon asymmetry without resonance, it is not necessary to think about so finely tuned condition. Again in the region where the resonant condition is applicable, the J_{CP} is miserably small to observe through any experiments.

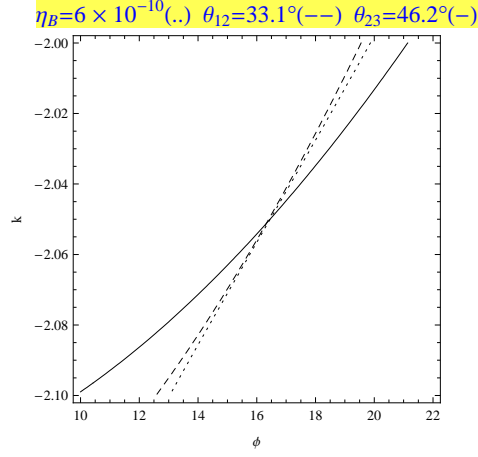


Figure 15: Contour plot of baryon asymmetry η_B , θ_{12} and, θ_{23} in ϕ - k plane for Δm_{32}^2 and Δm_{21}^2 to their best fit values and for mass scale of right handed neutrino $M_0 = 5 \times 10^{13}$ GeV.

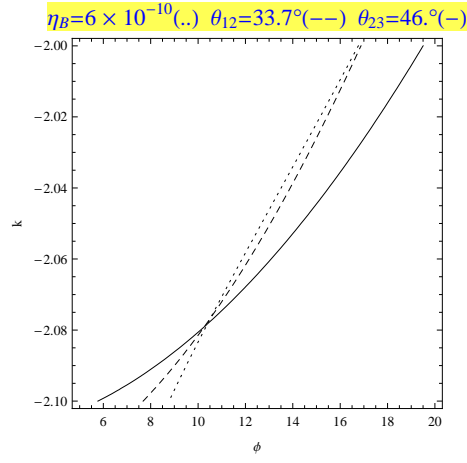


Figure 16: Contour plot of baryon asymmetry η_B , θ_{12} and, θ_{23} in ϕ - k plane for Δm_{32}^2 and Δm_{21}^2 to their best fit values and for mass scale of right handed neutrino $M_0 = 10^{14}$ GeV.

4.3 Final Analysis

We end our analysis with the help of three contour plots of baryon asymmetry $\eta_B = 6 \times 10^{-10}$ for three scales $M_0 = 2 \times 10^{13}$ GeV, $M_0 = 5 \times 10^{13}$ GeV and, $M_0 = 10^{14}$ GeV in ϕ - k plane. We insert the contours of θ_{23} and θ_{12} and manage to find the intersection of three contours for some reasonable value of θ_{23} and θ_{12} .

Case(I): $M_0 = 2 \times 10^{13}$ GeV, from Fig. 14, we are seeing that three contours $\theta_{12} = 31.86^\circ$, $\theta_{23} = 47^\circ$ and, $\eta_B = 6 \times 10^{-10}$ are intersecting at a point (20.5, -2.082) in ϕ - k plane. So, the mixing angles are within nearly 2° variation about the best fit values. Obtained k , ϕ value gives $\theta_{13} = 1.78^\circ$, $J_{CP} = 0.5 \times 10^{-2}$, $\delta_{CP} = 46^\circ$, $m_{\text{eff}} = 0.02423$ eV, $m_0 = 0.05365$ eV, $\epsilon = -0.121$ and $k + 2 \cos \phi = -0.208$.

Case (II): $M_0 = 5 \times 10^{13}$ GeV, now from Fig. 15, we have the intersection of the contour $\eta_B = 6 \times 10^{-10}$ with the contours $\theta_{12} = 33.1^\circ$, $\theta_{23} = 46.2^\circ$ at (16.0, -2.07) in ϕ - k plane. So, θ_{12} and θ_{23} are more closer to their best fit values (nearly 1° deviation from their best fit value). At this point we have $\theta_{13} = 1^\circ$, $J_{CP} = 0.8 \times 10^{-3}$, $\delta_{CP} = 12^\circ$, $m_{\text{eff}} = 0.02175$ eV, $m_0 = 0.0516$ eV, $\epsilon = -0.076$ and $k + 2 \cos \phi = -0.131$.

Case (III): $M_0 = 10^{14}$ GeV, from the Fig. 16, the contours $\eta_B = 6 \times 10^{-10}$, $\theta_{12} = 33.7^\circ$ and, $\theta_{23} = 46.0^\circ$ have crossed in ϕ - k plane at (10.5, -2.08). So, higher scale of right handed neutrino mass helps to have very good value of θ_{12} and θ_{23} . At the intersection point, we get $\theta_{13} = 0.8^\circ$, $J_{CP} = 0.23 \times 10^{-3}$, $\delta_{CP} = 4.5^\circ$, $m_{\text{eff}} = 0.0194$ eV, $m_0 = 0.051$ eV, $\epsilon = -0.065$ and $k + 2 \cos \phi = -0.11$.

The small value of ϵ compatible with all experimental results. So, we can demand that A_4 is an approximate symmetry.

4.4 Corelation of CP violating phases in this model

In this model everything is determinable in terms of parameter k and ϕ . Well measured quantities fix the value of those parameters. So, value of the rest of the physical quantities (some of them not so well measured in experiment like θ_{13} , m_{eff} and some of them yet to measure in experiment like J_{CP} , δ_{CP} and the Majorana phases also) are obtainable in this model. Question may arise whether we can have any relations among the phases in this model or not. First of all, there are two kinds phases, low energy and high energy phases. High energy phases are responsible to generate the lepton asymmetry. The low energy phases are responsible for determining low energy leptonic CP violation. Low energy phases are in two type, one is the lepton no preserving CP violating phase δ_{CP} and another two are the lepton no breaking CP violating phases. In general, all phases are independent, meaning that there are no correlation among the phases between high and low energy sector, and also phases inside a particular sector are not correlated. For three generations

of neutrinos, there are three key phases which are responsible for leptogenesis. Those are phases in h_{12}^2 , h_{13}^2 , and h_{23}^2 . From matrix h given in Eq. (4.18), we have

$$\Phi_1 = \arg(h_{12}^2) = 2\alpha \quad \Phi_2 = \arg(h_{13}^2) = 2(\alpha - \gamma) \quad \Phi_3 = \arg(h_{23}^2) = -2\gamma. \quad (4.31)$$

It leads to the relation,

$$\Phi_2 = \Phi_1 + \Phi_3. \quad (4.32)$$

For a given k and ϕ the α and γ are known from Eq. (4.16). Hence Φ_1 , Φ_2 and Φ_3 are individually determinable phases. But, in this model values of those high energy phases follow the relation given in Eq. (4.32).

Now, let us give a fresh look to the leptonic mixing matrix. In Eq. (4.3) we have given the tri-bimaximal rotated form of the neutrino mass matrix. Keeping terms upto first order in ϵ we obtain the diagonalising matrix in the following form,

$$V' = U_{TB} \begin{pmatrix} 1 & -\epsilon X e^{-i\theta_X} & -\epsilon Y e^{-i\theta_Y} \\ \epsilon X e^{i\theta_X} & 1 & -\epsilon Z e^{-i\theta_Z} \\ \epsilon Y e^{i\theta_Y} & \epsilon X e^{i\theta_Z} & 1 \end{pmatrix} \quad (4.33)$$

where the X , Y , Z and the associated phases are completely known function of k and ϕ . An additional phase matrix V_P is needed to make masses of light neutrino real, from Eq. (4.3) we have the phase matrix

$$V_P = \begin{pmatrix} e^{i(\pi/2-\alpha)} & 0 & 0 \\ 0 & e^{i\pi/2} & 0 \\ 0 & 0 & e^{i(\pi/2-\gamma)} \end{pmatrix}. \quad (4.34)$$

To obtain the CKM form of mixing matrix we need to rotate V' by two diagonal phase matrix, let $U_{P1} = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ and $U_{P2} = \text{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$. So, we have

$$V' = U_{P1}^\dagger \{U_{P1} V' U_{P2}^\dagger\} U_{P2} = U_{P1}^\dagger V_{CKM} U_{P2} \quad (4.35)$$

Now the with θ_{13} small we can write

$$\begin{aligned} (V_{CKM})_{11} &= \cos \theta_{12} = V'_{11} e^{i(\alpha_1 - \beta_1)} = |V'_{11}| e^{i(\psi_1 + \alpha_1 - \beta_1)} \\ (V_{CKM})_{12} &= \sin \theta_{12} = V'_{12} e^{i(\alpha_1 - \beta_2)} = |V'_{12}| e^{i(\phi_1 + \alpha_1 - \beta_2)} \\ (V_{CKM})_{13} &= \sin \theta_{13} e^{-i\delta_{CP}} = V'_{13} e^{i(\alpha_1 - \beta_3)} = |V'_{13}| e^{i(\delta_1 + \alpha_1 - \beta_3)}, \end{aligned} \quad (4.36)$$

and more six relations. But these three are sufficient for our discussions. The phases associated to V' elements, like ψ_1 , ϕ_1 , and δ_1 associated to V'_{11} , V'_{12} and V'_{13} respectively, are completely

determinable in terms of k , ϕ using functional form of ϵ , X , Y , Z and their associates phases. Now from the Eq. (4.36) we obtain the following phase relations

$$\psi_1 + \alpha_1 - \beta_1 = 0, \quad \phi_1 + \alpha_1 - \beta_2 = 0, \quad \delta_1 + \alpha_1 - \beta_3 = -\delta_{CP}. \quad (4.37)$$

Now the form of total mixing matrix is,

$$\begin{aligned} V &= V'V_P = U_{P1}^\dagger V_{CKM} U_{P2} V_P = U_{P1}^\dagger V_{CKM} \begin{pmatrix} e^{i(\pi/2-\alpha+\beta_1)} & 0 & 0 \\ 0 & e^{i(\pi/2+\beta_2)} & 0 \\ 0 & 0 & e^{i(\pi/2-\gamma+\beta_3)} \end{pmatrix} \\ &= \{e^{i(\pi/2-\gamma+\beta_3)} U_{P1}^\dagger\} V_{CKM} \begin{pmatrix} e^{i(\beta_1-\beta_3+\gamma-\alpha)} & 0 & 0 \\ 0 & e^{i(\gamma+\beta_2-\beta_3)} & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned} \quad (4.38)$$

This phase part in the parenthesis can be absorbed to charged lepton fields and the remaining part gives the leptonic mixing matrix of the form $V_{PMNS} = V_{CKM} \times \text{diag}(e^{i\alpha_M}, e^{i\beta_M}, 1)$, where the α_M and β_M are the two Majorana phases of leptonic mixing matrix. From Eq. (4.38) and using relations in Eq. (4.37) and Eq. (4.31), we have the Majorana phases,

$$\alpha_M = -\delta_{CP} - \frac{\Phi_2}{2} + \Psi \quad \beta_M = -\delta_{CP} - \frac{\Phi_3}{2} + \Phi \quad (4.39)$$

where $\Psi = \psi_1 - \delta_1$ and $\Phi = \phi_1 - \delta_1$ are known function of k and ϕ . In Eq. (4.39) we have the relation of low energy and high energy phases. So, in our model we have correlation among the CP violating phases.

5 Conclusion

We have shown that non-zero U_{e3} is generated in a softly broken A_4 symmetric model through see-saw mechanism incorporating single parameter perturbation in m_D in single element. First, we have studied all possible nine cases to explore the mixing angles considering all model parameters real. The extent of θ_{13} investigated, keeping the experimental values of present solar and atmospheric mixing angles. Among all nine possible texture of m_D some of them generates non-zero θ_{13} . Out of those non-zero θ_{13} generating textures of m_D we find that breaking at '12' and '13' elements are encompassing the best values of θ_{12} and θ_{23} . However, the reach of θ_{13} in those cases are around 1° . Considering one of the parameter complex we extend our analysis with one of the most suitable texture of m_D with breaking at '13' element. We have calculated mixing angles and neutrino mass squared differences in terms four model parameters (m_0 , ϵ , k , ϕ). We restrict model parameters utilising the well measured quantities Δm_\odot^2 , Δm_{atm}^2 , θ_{12} and θ_{23} and we have obtained θ_{13} (upto 2°) and large J_{CP} ($\simeq 10^{-2}$) and $|(M_\nu)_{ee}|$ well below the present experimental upper bound. In addition

to that a large δ_{CP} is also obtained. Further study on leptogenesis is also done and the present WMAP value of baryon asymmetry is obtained for a right handed neutrino mass scale $M_0 \simeq 10^{13}$ GeV. In our model, we have seen that the phases responsible for the leptogenesis are correlated. We also find out the relations among low energy CP violating phases and the lepton asymmetry phases. Small A_4 symmetry breaking parameter ϵ , is sufficient to describe the all low energy neutrino data and high energy CP violation (leptogenesis). So, A_4 symmetry is an approximate symmetry.

A

Here we consider breaking of A_4 symmetry in all other entries of m_D . Case (i) is already discussed in the text.

(ii) Breaking at '11' element : In this case m_D is given by

$$m_D = \frac{fv_u}{\sqrt{2}} \begin{pmatrix} 1+\epsilon & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (\text{A.1})$$

The mass eigenvalues are

$$m_1 = -\frac{f^2 v_u^2}{2(D+A)} \left(1 + \frac{4\epsilon}{3}\right) \quad m_2 = -\frac{f^2 v_u^2}{2A} \left(1 + \frac{2\epsilon}{3}\right) \quad m_3 = -\frac{f^2 v_u^2}{2(D-A)} \quad (\text{A.2})$$

and the three mixing angles come out as

$$\sin \theta_{12} = \frac{1}{\sqrt{3}} + \frac{2(2+k)}{3\sqrt{3}k}\epsilon \quad \sin \theta_{23} = -\frac{1}{\sqrt{2}} \quad \sin \theta_{13} = 0 \quad (\text{A.3})$$

The solar and atmospheric mass differences and their ratio are

$$\begin{aligned} \Delta m_{\odot}^2 &= \frac{m_0^2}{3(1+k)^2} [3k(k+2) + 4\epsilon(k^2 + 2k - 1)] \\ \Delta m_{atm}^2 &= \frac{m_0^2}{3(1-k)^2} [3k(2-k) - 4\epsilon(k-1)^2] \\ R &= \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} = \frac{(k-1)^2}{(k+1)^2} \frac{[3k(k+2) + 4\epsilon(k^2 + 2k - 1)]}{[3k(2-k) - 4\epsilon(k-1)^2]} \end{aligned} \quad (\text{A.4})$$

The obtained expression for ϵ in terms of model parameter k and experimentally known R :

$$\epsilon = \frac{3k}{4(k-1)^2} \frac{[R(2-k)(k+1)^2 - (k-1)^2(k+2)]}{[R(k+1)^2 + k^2 + 2k - 1]} \quad (\text{A.5})$$

iii) Breaking at '33' element : In this case, the structure of m_D is given by

$$m_D = \frac{fv_u}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 + \epsilon \end{pmatrix}. \quad (\text{A.6})$$

Mass eigenvalues are

$$m_1 = -\frac{f^2 v_u^2}{2(D+A)} \left(1 + \frac{\epsilon}{3}\right) \quad m_2 = -\frac{f^2 v_u^2}{2A} \left(1 + \frac{2\epsilon}{3}\right) \quad m_3 = -\frac{f^2 v_u^2}{2(D-A)} (1 + \epsilon) \quad (\text{A.7})$$

and the angels are

$$\sin \theta_{12} = \frac{1}{\sqrt{3}} - \frac{2+k}{3\sqrt{3}k} \epsilon \quad \sin \theta_{23} = -\left[\frac{1}{\sqrt{2}} - \epsilon \frac{k(4-k)}{6\sqrt{2}(2-k)} \right] \quad \sin \theta_{13} = -\frac{\epsilon k(1-k)}{3\sqrt{2}(2-k)} \quad (\text{A.8})$$

The solar and atmospheric mass differences and their ratio are

$$\begin{aligned} \Delta m_{\odot}^2 &= \frac{m_0^2}{3(1+k)^2} [3k(k+2) + 2\epsilon(2k^2 + 4k + 1)] \\ \Delta m_{atm}^2 &= \frac{m_0^2}{3(1-k)^2} [3k(2-k) - 2\epsilon(2k^2 - 4k - 1)] \\ R &= \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} = \frac{(k-1)^2 [3k(k+2) + 2\epsilon(2k^2 + 4k + 1)]}{(k+1)^2 [3k(2-k) - 2\epsilon(2k^2 - 4k - 1)]} \end{aligned} \quad (\text{A.9})$$

The obtained expression for ϵ in terms of model parameter k and experimentally known R :

$$\epsilon = \frac{3k}{2} \frac{[R(2-k)(k+1)^2 - (k-1)^2(k+2)]}{[(k-1)^2(2k^2 + 4k + 1) + R(k+1)^2(2k^2 - 4k - 1)]} \quad (\text{A.10})$$

iv) Breaking at '12' element : In this case, the structure of m_D is given by

$$m_D = \frac{fv_u}{\sqrt{2}} \begin{pmatrix} 1 & \epsilon & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\text{A.11})$$

Mass eigenvalues are

$$m_1 = -\frac{f^2 v_u^2}{2(D+A)} \left(1 - \frac{2\epsilon}{3}\right) \quad m_2 = -\frac{f^2 v_u^2}{2A} \left(1 + \frac{2\epsilon}{3}\right) \quad m_3 = -\frac{f^2 v_u^2}{2(D-A)} \quad (\text{A.12})$$

and the three mixing angles come out as

$$\sin \theta_{12} = \frac{1}{\sqrt{3}} + \frac{2k+1}{3\sqrt{3}k} \epsilon \quad \sin \theta_{23} = -\left[\frac{1}{\sqrt{2}} - \epsilon \frac{k(1-k)}{6\sqrt{2}(2-k)} \right] \quad \sin \theta_{13} = -\frac{\epsilon}{3\sqrt{2}} \frac{k^2 - k - 3}{(k-2)} \quad (\text{A.13})$$

The solar and atmospheric mass differences and their ratio are

$$\begin{aligned}\Delta m_{\odot}^2 &= \frac{m_0^2}{3(1+k)^2} [3k(k+2) + 4\epsilon(k^2 + 2k + 2)] \\ \Delta m_{atm}^2 &= \frac{m_0^2}{3(1-k)^2} [3k(2-k) - 4\epsilon(1-k)^2] \\ R &= \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} = \frac{(k-1)^2}{(k+1)^2} \frac{[3k(k+2) + 4\epsilon(k^2 + 2k + 2)]}{[3k(2-k) - 4\epsilon(1-k)^2]}\end{aligned}\quad (\text{A.14})$$

The obtained expression for ϵ in terms of model parameter k and experimentally known R :

$$\epsilon = \frac{3k}{4(1-k)^2} \frac{[R(2-k)(k+1)^2 - (k-1)^2(k+2)]}{[(k^2 + 2k + 2) + R(k+1)^2]} \quad (\text{A.15})$$

v) Breaking at '13' element : In this case, the structure of m_D is given by

$$m_D = \frac{fv_u}{\sqrt{2}} \begin{pmatrix} 1 & 0 & \epsilon \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\text{A.16})$$

The mass eigenvalues are

$$m_1 = -\frac{f^2 v_u^2}{2(D+A)} \left(1 - \frac{2\epsilon}{3}\right) \quad m_2 = -\frac{f^2 v_u^2}{2A} \left(1 + \frac{2\epsilon}{3}\right) \quad m_3 = -\frac{f^2 v_u^2}{2(D-A)} \quad (\text{A.17})$$

and the three mixing angles come out as

$$\begin{aligned}\sin \theta_{12} &= \frac{1}{\sqrt{3}} + \frac{2k+1}{3\sqrt{3k}} \epsilon & \sin \theta_{23} &= -\left[\frac{1}{\sqrt{2}} + \epsilon \frac{k(1-k)}{6\sqrt{2}(2-k)} \right] \\ \sin \theta_{13} &= -\frac{\epsilon}{3\sqrt{2}} \frac{k^2 - k - 3}{(k-2)}\end{aligned}\quad (\text{A.18})$$

The solar and atmospheric mass differences and their ratio are

$$\begin{aligned}\Delta m_{\odot}^2 &= \frac{m_0^2}{3(1+k)^2} [3k(k+2) + 4\epsilon(k^2 + 2k + 2)] \\ \Delta m_{atm}^2 &= \frac{m_0^2}{3(1-k)^2} [3k(2-k) - 4\epsilon(1-k)^2] \\ R &= \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} = \frac{(k-1)^2}{(k+1)^2} \frac{[3k(k+2) + 4\epsilon(k^2 + 2k + 2)]}{[3k(2-k) - 4\epsilon(1-k)^2]}\end{aligned}\quad (\text{A.19})$$

The obtained expression for ϵ in terms of model parameter k and experimentally known R :

$$\epsilon = \frac{3k}{4(1-k)^2} \frac{[R(2-k)(k+1)^2 - (k-1)^2(k+2)]}{[(k^2 + 2k + 2) + R(k+1)^2]} \quad (\text{A.20})$$

vi) Breaking at '23' element : In this case, the structure of m_D is given by

$$m_D = \frac{fv_u}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & 0 & 1 \end{pmatrix} \quad (\text{A.21})$$

The mass eigenvalues are

$$m_1 = -\frac{f^2 v_u^2}{2(D+A)} \left(1 + \frac{\epsilon}{3}\right) \quad m_2 = -\frac{f^2 v_u^2}{2A} \left(1 + \frac{2\epsilon}{3}\right) \quad m_3 = -\frac{f^2 v_u^2}{2(D-A)} (1 - \epsilon) \quad (\text{A.22})$$

and the three mixing angles come out as

$$\sin \theta_{12} = \frac{1}{\sqrt{3}} - \frac{\epsilon}{3\sqrt{3}} \frac{2+k}{k} \quad \sin \theta_{23} = -\left[\frac{1}{\sqrt{2}} - \frac{\epsilon}{2\sqrt{2}} \right] \quad \sin \theta_{13} = 0 \quad (\text{A.23})$$

The solar and atmospheric mass differences and their ratio are

$$\begin{aligned} \Delta m_{\odot}^2 &= \frac{m_0^2}{3(1+k)^2} [3k(k+2) + 2\epsilon(2k^2 + 4k + 1)] \\ \Delta m_{atm}^2 &= \frac{m_0^2}{3(1-k)^2} [3k(2-k) - 2\epsilon(2k^2 - 4k + 5)] \\ R &= \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} = \frac{(k-1)^2 [3k(k+2) + 2\epsilon(2k^2 + 4k + 1)]}{(k+1)^2 [3k(2-k) - 2\epsilon(2k^2 - 4k + 5)]} \end{aligned} \quad (\text{A.24})$$

The obtained expression for ϵ in terms of model parameter k and experimentally known R :

$$\epsilon = \frac{3k}{2} \frac{[R(2-k)(k+1)^2 - (k-1)^2(k+2)]}{[R(1+k)^2(2k^2 - 4k + 5) + (k-1)^2(2k^2 + 4k + 1)]} \quad (\text{A.25})$$

vii) Breaking at '21' element : In this case, the structure of m_D is given by

$$m_D = \frac{fv_u}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ \epsilon & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (\text{A.26})$$

The mass eigenvalues are

$$m_1 = -\frac{f^2 v_u^2}{2(D+A)} \left(1 - \frac{2\epsilon}{3}\right) \quad m_2 = -\frac{f^2 v_u^2}{2A} \left(1 + \frac{2\epsilon}{3}\right) \quad m_3 = -\frac{f^2 v_u^2}{2(D-A)} \quad (\text{A.27})$$

and the three mixing angles come out as

$$\begin{aligned} \sin \theta_{12} &= \frac{1}{\sqrt{3}} + \frac{\epsilon}{3\sqrt{3}} \frac{1-k}{k} & \sin \theta_{23} &= -\left[\frac{1}{\sqrt{2}} + \frac{\epsilon}{6\sqrt{2}} \frac{k(k-1)}{(2-k)} \right] \\ \sin \theta_{13} &= \frac{\epsilon}{3\sqrt{2}} \frac{(3+k^2-4k)}{(2-k)} \end{aligned} \quad (\text{A.28})$$

The solar and atmospheric mass differences and their ratio are

$$\begin{aligned}\Delta m_{\odot}^2 &= \frac{m_0^2}{3(1+k)^2} [3k(k+2) + 4\epsilon(k^2 + 2k + 2)] \\ \Delta m_{atm}^2 &= \frac{m_0^2}{3(1-k)^2} [3k(2-k) - 4\epsilon(1-k)^2] \\ R &= \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} = \frac{(k-1)^2}{(k+1)^2} \frac{[3k(k+2) + 4\epsilon(k^2 + 2k + 2)]}{[3k(2-k) - 4\epsilon(1-k)^2]}\end{aligned}\quad (\text{A.29})$$

The obtained expression for ϵ in terms of model parameter k and experimentally known R :

$$\epsilon = \frac{3k}{4(1-k)^2} \frac{[R(2-k)(k+1)^2 - (k+2)(k-1)^2]}{[R(1+k)^2 + k^2 + 2k + 2]} \quad (\text{A.30})$$

viii) Breaking at '31' element : In this case, the structure of m_D is given by

$$m_D = \frac{fv_u}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \epsilon & 0 & 1 \end{pmatrix} \quad (\text{A.31})$$

The mass eigenvalues are

$$m_1 = -\frac{f^2 v_u^2}{2(D+A)} \left(1 - \frac{2\epsilon}{3}\right) \quad m_2 = -\frac{f^2 v_u^2}{2A} \left(1 + \frac{2\epsilon}{3}\right) \quad m_3 = -\frac{f^2 v_u^2}{2(D-A)} \quad (\text{A.32})$$

and the three mixing angles come out as

$$\begin{aligned}\sin \theta_{12} &= \frac{1}{\sqrt{3}} + \frac{\epsilon}{3\sqrt{3}} \frac{1-k}{k} & \sin \theta_{23} &= -\left[\frac{1}{\sqrt{2}} + \frac{\epsilon}{6\sqrt{2}} \frac{k(1-k)}{(2-k)} \right] \\ \sin \theta_{13} &= -\frac{\epsilon}{3\sqrt{2}} \frac{3+k^2-4k}{2-k}\end{aligned}\quad (\text{A.33})$$

The solar and atmospheric mass differences and their ratio are

$$\begin{aligned}\Delta m_{\odot}^2 &= \frac{m_0^2}{3(1+k)^2} [3k(k+2) + 4\epsilon(k^2 + 2k + 2)] \\ \Delta m_{atm}^2 &= \frac{m_0^2}{3(1-k)^2} [3k(2-k) - 4\epsilon(1-k)^2] \\ R &= \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} = \frac{(k-1)^2}{(k+1)^2} \frac{[3k(k+2) + 4\epsilon(k^2 + 2k + 2)]}{[3k(2-k) - 4\epsilon(1-k)^2]}\end{aligned}\quad (\text{A.34})$$

The obtained expression for ϵ in terms of model parameter k and experimentally known R :

$$\epsilon = \frac{3k}{4(1-k)^2} \frac{[R(2-k)(k+1)^2 - (k-1)^2(k+2)]}{[R(1+k)^2 + k^2 + 2k + 2]} \quad (\text{A.35})$$

ix) Breaking at '32' element : In this case, the structure of m_D is given by

$$m_D = \frac{fv_u}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \epsilon & 1 \end{pmatrix} \quad (\text{A.36})$$

The mass eigenvalues are

$$m_1 = -\frac{f^2 v_u^2}{2(D+A)} \left(1 + \frac{\epsilon}{3}\right) \quad m_2 = -\frac{f^2 v_u^2}{2A} \left(1 + \frac{2\epsilon}{3}\right) \quad m_3 = -\frac{f^2 v_u^2}{2(D-A)} (1 - \epsilon) \quad (\text{A.37})$$

and the three mixing angles come out as

$$\sin \theta_{12} = \frac{1}{\sqrt{3}} - \frac{\epsilon}{3\sqrt{3}} \frac{2+k}{k} \quad \sin \theta_{23} = -\left[\frac{1}{\sqrt{2}} + \frac{\epsilon}{2\sqrt{2}}\right] \quad \sin \theta_{13} = 0 \quad (\text{A.38})$$

The solar and atmospheric mass differences and their ratio are

$$\begin{aligned} \Delta m_{\odot}^2 &= \frac{m_0^2}{3(1+k)^2} [3k(k+2) + 2\epsilon(2k^2 + 4k + 1)] \\ \Delta m_{atm}^2 &= \frac{m_0^2}{3(1-k)^2} [3k(2-k) - 2\epsilon(2k^2 - 4k + 5)] \\ R &= \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} = \frac{(k-1)^2 [3k(k+2) + 2\epsilon(2k^2 + 4k + 1)]}{(k+1)^2 [3k(2-k) - 2\epsilon(2k^2 - 4k + 5)]} \end{aligned} \quad (\text{A.39})$$

The obtained expression for ϵ in terms of model parameter k and experimentally known R :

$$\epsilon = \frac{3k}{2} \frac{[R(2-k)(k+1)^2 - (k-1)^2(k+2)]}{[R(1+k)^2(2k^2 - 4k + 5) + (1-k)^2(2k^2 + 4k + 1)]} \quad (\text{A.40})$$

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